Solihull Sixth Form College
Further Maths
Summer Homework

The purpose of this exercise is to give you an idea of the work we do in A-level Further Maths. The topic being tested here is Complex Numbers, in which we extend the set of numbers to include not only positive and negative integers fractions and irrational numbers, but also square roots of negative numbers, which have been previously forbidden. We define i to be the number that you get when you square root -1, so $i = \sqrt{-1}$. This leads to $i^2 = -1$, and allows us to write the square root of any negative in terms of i, as $\sqrt{-4} = \sqrt{4}\sqrt{-1} = 2i$. Other than this, i works like any other number, so $i + i = 2i$ and $2(1 + 3i) = 2 + 6i$

We've written these questions to challenge you, so don't worry if they're a little hard: the aim is to explore a novel bit of mathematics that you might not have seen before. We'll cover all the skills you need to do these questions in the first couple of weeks of your Further Maths course, but if you want to do some extra research, or you need a hint on the exercise, then have a look at the following videos:

TL Maths: B1: Introducing Complex Numbers:

https://sites.google.com/view/tlmaths/home/a-level-further-maths/pure/b-complex-numbers/b1-introducing-complex-numbers

TL Maths B2: Working with Complex Numbers:

https://sites.google.com/view/tlmaths/home/a-level-further-maths/pure/b-complex-numbers/b2-working-with-complex-numbers

We hope you find the following questions interesting, and look forward to seeing you in September.

Mathematics Department

Solihull Sixth Form College

Exercise

1) Solve the following quadratic equations.

Give your answers in terms of i.

1a) $x^2 + 64 = 0$

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1b) $x^2 - 6x + 73 = 0$

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2) Write each of the following expressions in the form $a + bi$.

2a) $(3 + 6i) + i(7 - i)$

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2b) $(2 + 3i)(1 - i)$

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2c) $(4 - 2i)^2$

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3) a) Given that $z = 4 - i$, calculate the following, giving your answers in the form $a + bi$:

3ai) $8z$

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3aii) z^2

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3b) By substituting your answers to part a) into the equation, show that $z = 4 - i$ is a solution of the equation

$$z^2 - 8z + 17 = 0$$

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3c) Find the other solution to this equation, giving your answer in the form $a + bi$.

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4) The numbers p and q are such that

$$p + q = 2$$

$$pq = 10$$

Find the value of p and the value of q.

Give your answers in terms of i.

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5 The equation $x^2 + kx + 40 = 0$, where k is a constant, has a solution $x = -2 + 6i$

5a) Show, by substituting into the equation, that $k = 4$.

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5b) Find the other solution to this equation.

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6) a) Given that $i^2 = -1$, find, in terms of i where appropriate, the following:

6ai) i^3

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6aii) i^4

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6aiii) i^{2021}

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6b) Explain why $i^{-1} = i^3$

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6c) What numbers, n, solve the equation $i^n = -1$?

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7) Rearrange each of the following to find the value of z. Give your answers in the form $a + bi$.

7a) $2(z - i) = 4 + 3i - z$

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7b) $\frac{3z+2}{2i-z} = \frac{4}{3}$

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