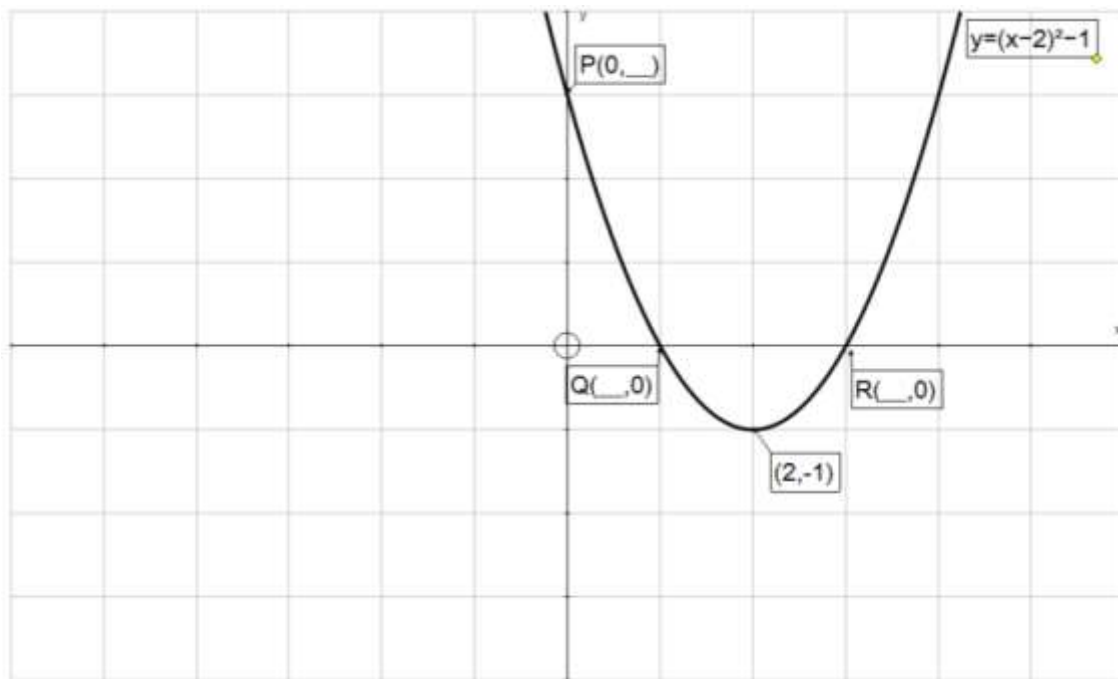


Further Maths Summer Homework

Name: _____

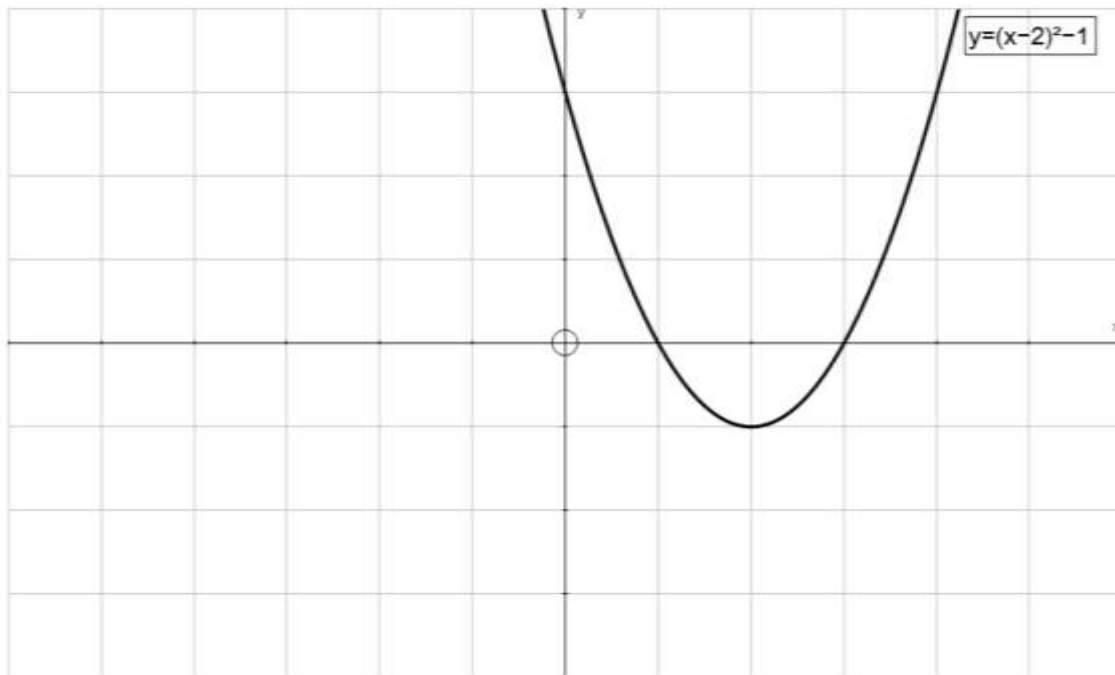
In your GCSE courses, you've doubtlessly come across a number of geometric transformations such as stretches, reflections and translations. At A-level, we discover how to extend these ideas to transforming curves and their equations, and, in the Further Maths course, we find out how to encode these transformations in a very elegant way using matrices.

- 1) The sketch below shows the graph of $y = (x-2)^2 - 1$. The minimum point of this graph has coordinates $(2, -1)$.



- a) What is the y-coordinate of point P, where the graph cuts the y-axis?
- b) What are the x-coordinates of points Q and R, where the graph cuts the x-axis?
- c) On the same axes above, sketch the graph of $y = x^2$
- d) Describe the geometric transformations which map the graph of $y = x^2$, that you've just drawn, onto the printed graph of $y = (x-2)^2 - 1$.

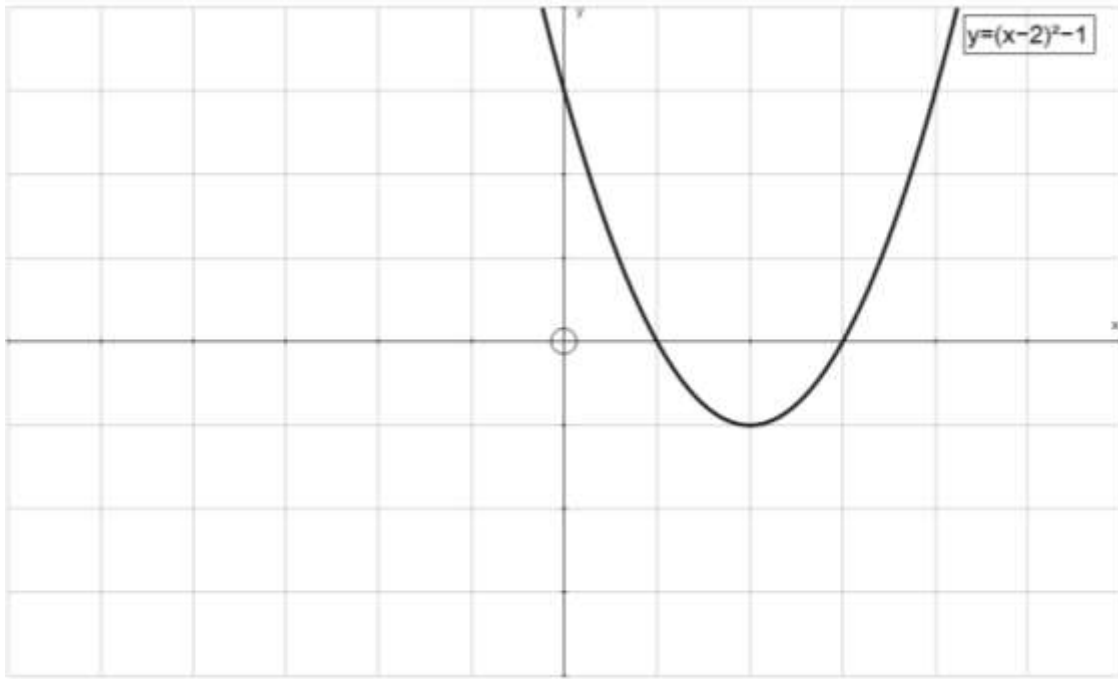
- 2) The sketch, below, is another copy of $y = (x-2)^2 - 1$:



The graph of $y = (x+1)^2 - 1$ is obtained by translating each of the points on $y = (x-2)^2 - 1$ by vector $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$.

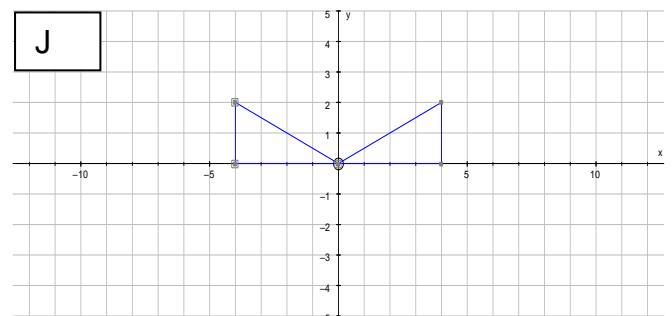
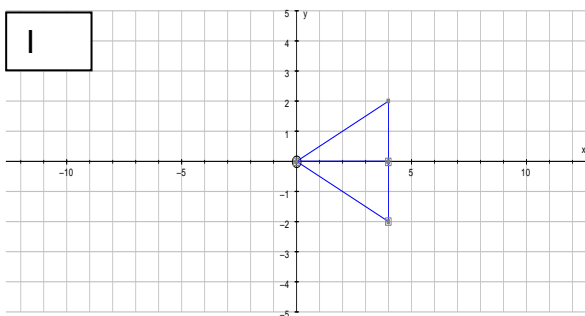
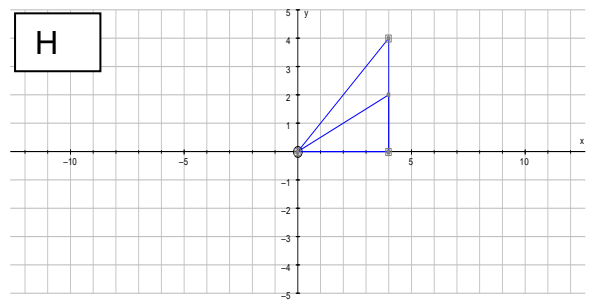
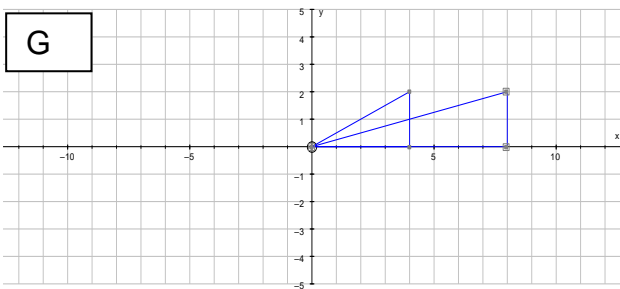
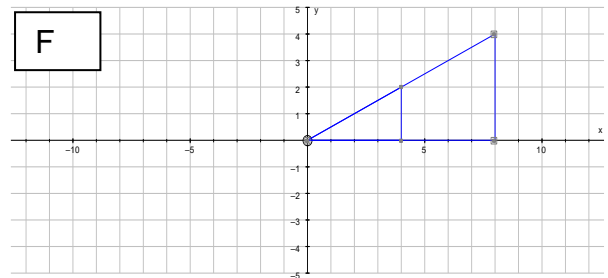
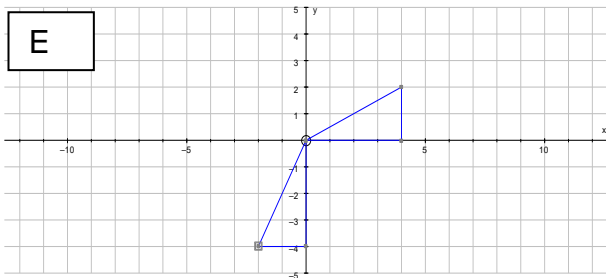
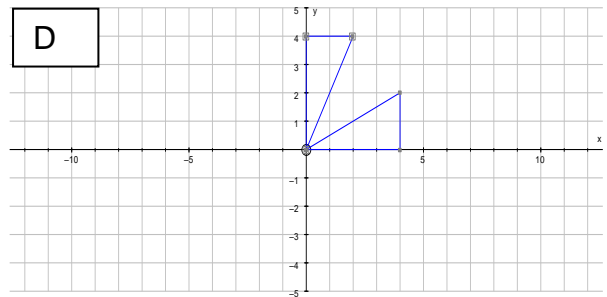
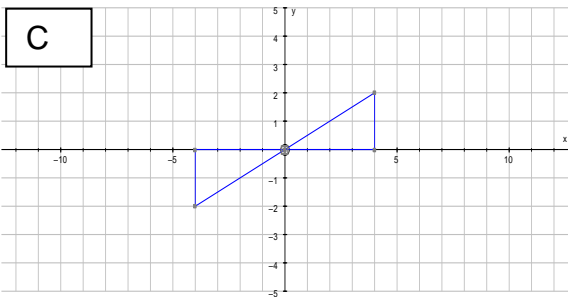
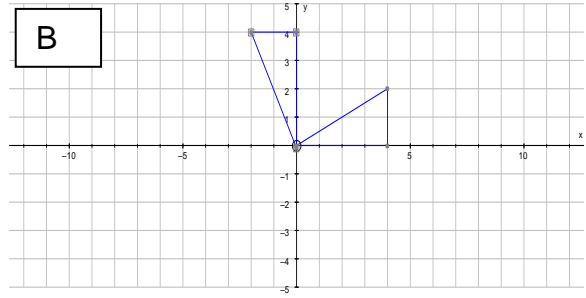
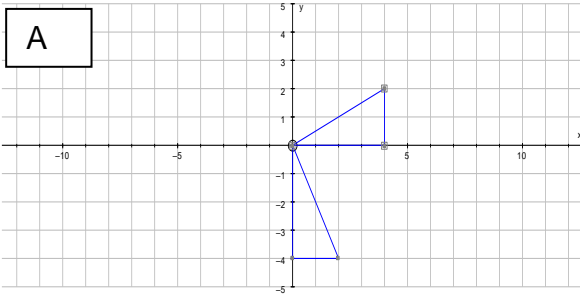
- a) The minimum point of $y = (x-2)^2 - 1$ has coordinates $(2, -1)$. What are the coordinates of the minimum point of $y = (x+1)^2 - 1$?
- b) What are the coordinates of the points where $y = (x+1)^2 - 1$ cuts the x-axis?
- c) Sketch and label the graph of $y = (x+1)^2 - 1$ on the axes above.
- 3) The graph of $y = (x-1)^2 + 1$ is a translation of $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ applied to our original graph.
- a) Sketch and label the graph of $y = (x-1)^2 + 1$ on the axes above.
- b) How does your sketch imply that the equation $0 = (x-1)^2 + 1$ has no real solutions?

4) Here's another copy of $y = (x-2)^2 - 1$:



- ai) The graph of $y = (x+2)^2 - 1$ is a reflection of $y = (x-2)^2 - 1$ in the y-axis. Sketch and label $y = (x+2)^2 - 1$ on the axes, above.
- aii) Label the coordinates of the minimum point of $y = (x+2)^2 - 1$ on your sketch.
- bi) The graph of $y = \left(\frac{1}{2}x - 2\right)^2 - 1$ is a stretch by scale factor 2 parallel to the x-axis of the curve with equation $y = (x-2)^2 - 1$. Sketch and label $y = \left(\frac{1}{2}x - 2\right)^2 - 1$ on the axes above.
- bii) Label the coordinates of the minimum point of $y = \left(\frac{1}{2}x - 2\right)^2 - 1$ on your sketch.
- c) State the equation of the invariant line of the two transformations described in parts a) and b). [An invariant line is one which isn't changed by the transformation.]

5) On the next page are ten triangles and their images under a transformation, labelled A-J. Match each one with the relevant description, labelled 1-10 and transformation matrix, labelled i) to x), from the following two pages.



1. Rotation by 90° clockwise
2. Rotation by 90° anticlockwise
3. Reflection in y -axis
4. Reflection in x -axis
5. Reflection in the line $y = x$
6. Reflection in the line $y = -x$
7. Rotation by 180°
8. Enlargement with scale factor 2, centre the origin
9. Stretch with scale factor 2, in the direction of the x -axis
10. Stretch with scale factor 2, in the direction of the y -axis

i) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	ii) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
iii) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	iv) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
v) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	vi) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$
vii) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	viii) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$
ix) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	x) $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$